Group decision making models based on trapezoidal vague sets and its application in coal mine

Xiaoguo Chen^{2,4,5}, Dan Yang², Yue Yang^{3,4}, Minghui Peng²

Abstract. The emergency rescue capability evaluation method of coal mine is discussed. The three group decision making models are established based on the score value, utility value and VIKOR method of trapezoidal vague set theory. The weights of experts and all levels indexes are calculated by the improved vague entropy which is proposed in this paper. On the basis of this, trapezoidal vague value weighted count average operator is used to integrate the second level index vague value evaluation matrix which is given by all the experts, and evaluation decision making is made respectively by calculating the score value, utility value and benefit ratio value of different schemes. The above three models are successively applied in four related coal mines of Datong Coal Mine Group. The strongest and the worst emergency rescue capabilities of coal mine obtained based on the three decision making models are of full agreement. The ranking of emergency rescue capabilities of coal mine obtained based on trapezoidal vague value score value and VIKOR method are the same.

Key words. Safety warning of coal mine, emergency rescue capability evaluation, and vikor decision making model, trapezoidal vague value integrated operational.

1. Introduction

Coal is one of the main sources of energy in China in modern times, along with the coal mining, the coal mine accidents have not been eliminated [1]. Coal mine

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 $^{^2 \}rm Workshop$ 1 - College of Science, Heilongjiang University of Science & Technology, Harbin 150022, China

³Workshop 2 - School of Civil Engineering, Heilongjiang University of Science & Technology, Harbin 150022, China; e-mail: 2321120911@qq.com

⁴Workshop 3 - Heilongjiang Ground Pressure & Gas Control in Deep Mining Key Lab, Heilongjiang University of Science & Technology, Harbin 150022, China; e-mail: yybeijing@126.com

⁵Corresponding author: Xiaoguo Chen; e-mail: kjdxcxg@163.com

accidents not only threaten the life and health of miners and make coal mining enterprises suffer economic losses, but also cause bad social impact [2]. Many scholars have done a lot of research on the evaluation of coal mine emergency rescue capacity [3-6]. Szmidt [7] established the evaluation method of coal mine emergency rescue capability by using the fuzzy gravity center method on the basis of constructing index system. Luca [8] determined the index weight by rough set theory, and established the coal mine emergency rescue capability evaluation system based on extension model. Xu [9] used interval valued intuitionistic fuzzy entropy to determine the weight of each index factor, and established the evaluation model of coal mine emergency rescue capability based on interval valued intuitionistic fuzzy set theory, then evaluated the emergency rescue capability of different coal mines. Xia [10] integrated the improved analytic hierarchy process (AHP), entropy weight method and fuzzy comprehensive evaluation method, then established the coal mine emergency rescue capability evaluation model.

However, the above research results are mostly based on fuzzy sets. Since the fuzzy set in the actual application only consider the membership information of things without taking the non membership degree into account, it has led to the insufficient description of things and restricted the reliability of research results. In order to make up for the above shortcomings, W.L Gau et al. (1993) proposed the concept of Vague set and introduced non membership degree which made the theory widely used in decision making field. However, there are few studies on the trapezoidal vague set in the existing achievements, especially in the aspect of integrated computing. Based on this, this paper will continue to explore the concept of the trapezoidal vague set distance, the scores, uncertainty and utility function, then propose the trapezoidal vague number weighted arithmetic average operator. We will study their excellent properties, and on the basis of them, we will establish several coal mine emergency rescue capability evaluation models of group decision making. Finally, the proposed theory will be applied in some mines of Datong Coal Mine Group, and the results of the experiment are analysed.

2. Text basic theory of vague sets

2.1. vague set

Definition and the operation rules of vague sets were given by the reference [3].

Let C_{43} be a universe of discourse. Two mappings on the vague set C_{45} in B_4 are $C_{48}; C_{51}$, with the condition C_{52} , where C_{53} and B_4 denote the degree of membership and non-membership which support and oppose the evidence of C_{54} respectively; Y_i is called the hesitancy degree. The greater the P_1 is, the less the information of C_{12} we know. When C_{13} , vague sets degenerate into fuzzy sets . Generally, the degree of C_{14} belonging to vague set C_{12} is denoted as interval Y_2 .

2.2. Trapezoidal Vague number

Let A be a vague set in universe of discourse U, a is a Vague number of A, if

$$t_{\tilde{\alpha}}(u) \begin{cases} \frac{u-a}{b-a}t_{\tilde{\alpha}}, & a \le u \le b; \\ t_{\tilde{\alpha}}, & b \le u \le c; \\ \frac{d-u}{d-c}t_{\tilde{\alpha}}, & c \le u \le d; \\ 0, & other. \end{cases}$$
(1)

$$f_{\tilde{\alpha}}(u) \begin{cases} \frac{b-u+(u-a_{1})f_{\tilde{\alpha}}}{b-a_{1}}, & a_{1} \leq u \leq b; \\ f_{\tilde{\alpha}}, & b \leq u \leq c; \\ \frac{u-c+(d_{1}-u)f_{\tilde{\alpha}}}{d_{1}-c}, & c \leq u \leq d_{1}; \\ 0, & other. \end{cases}$$
(2)

where $0 \le t \le 1, 0 \le f \le 1$, and $a1 \le a \le b \le c \le d \le d1$, then we call

$$\tau = \tau_0 \left(1/2 - \xi \right) \,, \tag{3}$$

a trapezoidal vague number.

Usually we let [a,b,c,d] = [a1,b,c,d1], so a = ([a,b,c,d]). The trapezoidal vague numbers that we study in this paper are all defined in this case in order to facilitate the study we assume a>0.

Let a1=([a,b,c,d]), a2=(a2,b2,c2,d2) be two sets of trapezoidal vague numbers, then

(1)

$$E = E_0 \left(1 - \gamma \tau \right) \,, \tag{4}$$

(2)

$$E = E_0 \left(1 - \alpha \left(1/2 - \xi \right) \right) \,, \tag{5}$$

Let ai=([ai,bi,ci,di])be a set of trapezoidal vague numbers, w=(w1,w2,wn) is the weight set corresponding to 0 < wi < 1, then

$$h(\xi) = h_0 \left[1 - (1 - \beta_1) \left(\xi + 1/2 \right) \right] \cdot \left[1 - (1 - \beta_2) \left(\eta + 1/2 \right) \right], \tag{6}$$

is called the weighted arithmetic average operator of a.

3. Group decision making models based on trapezoidal vague sets

In a group decision making problem, let $P = \{p_1, p_2, \dots, p_l\}$ be an expert set, l is the number of the experts; $Y = \{Y_1, Y_2, \dots, Y_m\}$ be an case set, m is the number of the cases; $A = \{B_1, B_2, \dots, B_n\}$ be an factor set, n is the number of the factors.

 $F_k(A) = (a_{ij}(k))_{m \times n}$ is a trapezoidal vague number decision matrix that is given by the expert pk, where

 $a_{ij}(k)$ i $(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; k = 1, 2, \cdots, l)$ s the trapezoidal vague number corresponding to the j_{th}

Index in the case Yi which aimed by the k_{th} expert.

The expert weight is determined. Since each expert has different understanding of different cases, the expert weight for each case is different. Firstly, fix the case $Yi(i = 1, 2, \dots, m)$, bring the evaluation values of index B_j by expert pk into the formula

$$E_{B_j}^{(i)}(p_k) = \frac{1 - (t_{\alpha_{ij}(k)} - f_{\alpha_{ij}(k)})^2 + 2\pi_{\alpha_{ij}(k)}^2}{2 - (t_{\alpha_{ij}(k)} - f_{\alpha_{ij}(k)})^2 + \pi_{\alpha_{ij}(k)}^2}.$$
(7)

Then the entropy $E_{B_i}^{(i)}(p_k)$ corresponding to the expert is obtained.

Next we get the weight of expert pk on Index B_j under the case Yi by formula

$$w_{B_j}^{(i)}(p_k) = \frac{1 - E_{B_j}^{(i)}(p_k)}{\sum_{k=1}^l \left(1 - E_{B_j}^{(i)}(p_k)\right)}.$$
(8)

Determination of index weight. First we calculate the vague entropy of the j_{th} index in $\tilde{F}(A) = (a_{ij})_{m \times n}$ by

$$E(B_j) = \frac{1}{m} \sum_{i=1}^m \frac{1 - (t_{\alpha_{ij}} - f_{\alpha_{ij}})^2 + 2\pi_{\alpha_{ij}}^2}{2 - (t_{\alpha_{ij}} - f_{\alpha_{ij}})^2 + \pi_{\alpha_{ij}}^2}.$$
(9)

Then we get the weight of the j_{th} index by

$$w_j = \frac{1 - E(B_j)}{\sum_{j=1}^n (1 - E(B_j))}.$$
(10)

3.1. Group decision making method based on the score of trapezoidal vague numbers.

The decision method is mainly based on the trapezoidal vague number score, let the evaluation value which is trapezoidal vague number of index B_j of case $\operatorname{Yi}(i = 1, 2, \dots, m)$ be $\alpha_{ij} = ([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; t_{\alpha_{ij}}, 1 - f_{\alpha_{ij}}).$

We can rank it according to the score value $S(\alpha_{ij}) = (t_{\alpha_{ij}} - f_{\alpha_{ij}})(a_{ij} + b_{ij} + c_{ij} + d_{ij})$ and the exact value

 $H(\alpha_{ij}) = (t_{\alpha_{ij}} + f_{\alpha_{ij}})(a_{ij} + b_{ij} + c_{ij} + d_{ij})$ when we make decisions, if $S(\alpha_{ij}(1)) > S(\alpha_{ij}(2))$, then

 $\alpha_{ij}(1) > \alpha_{ij}(2); \text{ If } S(\alpha_{ij}(1)) = S(\alpha_{ij}(2)), H(\alpha_{ij}(1)) > H(\alpha_{ij}(2)), \text{ then}\alpha_{ij}(1) > \alpha_{ij}(2); \text{ If } S(\alpha_{ij}(1)) = S(\alpha_{ij}(2)), H(\alpha_{ij}(1)) = H(\alpha_{ij}(2)), \text{ then}\alpha_{ij}(1) = \alpha_{ij}(2).$

Group decision making steps:

(1) Calculate each expert weight $w_{B_j}^{(i)}$ of different index under different coal mine by formula (8);

(2) Aggregate all the trapezoidal vague numbers that were given by expert by the integral operator formula

(3) Determine the attribute weight w_j of each index by formula (10);

(4) Calculate the score s_{ij} $(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ of the integrated data by the score function of trapezoidal Vague number;

(5) Calculate the final score of every case by the formula $s_i = \sum_{j=1}^n w_j s_{ij} (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n);$

(6) Make a sequence based on the score of every case according to the principle that it will be better if the score is greater.

3.2. Group decision making method based on utility number of trapezoidal vague numbers.

Let the evaluation value which is trapezoidal vague number of index B_j of case $Yi(i = 1, 2, \dots, m)$ be

$$\alpha_{ij} = \left([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; t_{\alpha_{ij}}, 1 - f_{\alpha_{ij}} \right) \cdot \Delta(\alpha_{ij}) = \frac{a_{ij} + b_{ij} + c_{ij} + d_{ij}}{4} (1 - t_{\alpha_{ij}} - f_{\alpha_{ij}})$$

denotes the unknown degree of the trapezoidal vague number,

$$\sigma(\alpha_{ij}) = \frac{a_{ij} + b_{ij} + c_{ij} + d_{ij}}{4} (1 - |t_{\alpha_{ij}} - f_{\alpha_{ij}}|)$$

denotes the fuzziness degree of the trapezoidal vague number,

$$r(\alpha_{ij}) = \Delta(\alpha_{ij}) + \sigma(\alpha_{ij})$$

denotes the uncertainty degree of the trapezoidal vague number.

$$u(\alpha_{ij}) = \frac{1}{1 + e - \frac{a_{ij} + b_{ij} + c_{ij} + d_{ij}}{4} \left(t_{\alpha_{ij}} - f_{\alpha_{ij}} \right) \cdot \left(1 - r(\alpha_{ij}) \right)}$$

is called the actual utility function of α_{ij} ,

A is called the chance utility function of α_{ij} , $m(\alpha_{ij}) = u(\alpha_{ij}) + v(\alpha_{ij})$ is called the utility function of α_{ij} .

Group decision making steps:

(1) Calculate the utility value $m(\alpha_{ij})$ $(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ of the integrated data by the utility function of trapezoidal vague number;

(2) calculate the final utility value of every case by the formula $m(\alpha_i) = \sum_{j=1}^n w_j m(\alpha_{ij}) (i = 1, 2, \cdots, m);$

(3) Make a sequence based on the utility value of every case according to the principle that it will be better if the value is greater.

3.3. VIKOR decision making method based on trapezoidal vague numbers.

The negative and positive ideal solutions α and α_{ij} are

$$\rho = \rho_0 \left[1 - (1 - \beta) \left(\xi + 1/2 \right)^2 \right], \tag{11}$$

where $\alpha = \gamma \tau_0$ ($0 \le \alpha \le 1$), I_1 denotes the utility attribute, I_2 denotes the cost

attribute.

$$T = \frac{ab}{2}\omega^2 \int_A h(\xi)\rho w^2 \,\mathrm{d}A \tag{12}$$

$$V = \frac{ab}{2} \int_{A} D(\xi) [G - 2(1 - \nu)H] \,\mathrm{d}A\,, \tag{13}$$

Where w_j is the j_{th} attribute weight. When S_i is smaller, the corresponding group utility value is larger. And when R_i smaller and corresponding individual regret value is smaller.

Then we use S_i and R_i to determine the benefit ratio Q_i of every case.

$$D(\xi) = D_0 \left[\left[1 - (1 - \beta_1) \left(\xi + 1/2 \right) \right] \cdot \left[1 - (1 - \beta_2) \left(\eta + 1/2 \right) \right] \right]^3, \tag{14}$$

Where α denotes the coefficient of decision making mechanism, then it shows that group utility plays a leading role in decision making process; If $\alpha > 0.5$, then it shows that individual regret value plays a leading role in decision making process; If $\alpha < 0.5$, then it shows that decision making is achieved through a balanced compromise.

Group decision making steps:

(1) Determine positive and negative ideal solutions Q_i and α in the case set by formula (11);

(2) Calculate the group utility value S_i , the individual regret value R_i and the benefit ratio Q_i by formula (12), (13) and (14);

(3) Make decisions according to the principle that it will be better if the Q_i is greater.

4. Application

According to the actual background of Datong Coal Mine Group, the evaluation index system of coal mine emergency rescue capability is determined, it is made up of 5 factors, the results are shown in Table 1.

Elevation of Rain-Distance from the Central At mo-Rainfall within Six fall Observation Center spheric Pressure Hours 54049095017.2540216950131.8540950 111168.880 482950 19.9

Table 1. The emergency rescue capability evaluation index system of DaTong coal mine group

We evaluate the emergency rescue capability of the four coal mines of Datong Coal Mine Group.

The results of the three experts Pk on the above 5 factors are expressed in the

	B_1	B_2	B_3	B_4	B_5
Y_1	([1,2,4,6]; 0.5,0.6)	([2,4,5,6]; 0.3,0.4)	([2,3,5,6]; 0.3,0.4)	([1,2,5,6]; 0.4,0.5)	([2,3,4,6]; 0.5,0.6)
Y_2	([2,3,5,6]; 0.4,0.6)	([3,4,6,7]; 0.5,0.7)	([2,3,4,5]; 0.2,0.4)	([3,4,6,7]; 0.7,0.8)	([2,3,4,5]; 0.6,0.8)
Y_3	([3,4,5,6]; 0.3,0.6)	([1,2,4,5]; 0.4,0.5)	([1,2,3,5]; 0.5,0.6)	([4,5,6,7]; 0.4,0.6)	([1,2,3,4]; 0.6,0.7)
Y_4	([1,3,4,5]; 0.7,0.8)	([1,2,3,4]; 0.1,0.4)	([2,3,4,5]; 0.3,0.4)	([1,2,3,4]; 0.2,0.3)	([3,4,5,6]; 0.7,0.8)

form of trapezoidal Vague numbers, shown in Table 2-4.

Table 2. The trapezoidal vague number evaluation information of P_1

Table 3. The trapezoidal vague number evaluation information of P_2

	B_1	B_2	B_3	B_4	B_5
Y_1	([1,3,4,5]; 0.4,0.7)	([2,3,4,5]; 0.2,0.3)	([2,3,4,6]; 0.3,0.4)	([1,2,3,6]; 0.4,0.5)	([2,3,4,5]; 0.2,0.6)
Y_2	([2,3,4,6]; 0.3,0.8)	([3,4,6,7]; 0.6,0.9)	([1,2,4,5]; 0.2,0.4)	([3,4,5,7]; 0.6,0.8)	([1,3,4,5]; 0.3,0.8)
Y_3	([2,4,5,6]; 0.5,0.6)	([1,2,4,6]; 0.4,0.7)	([2,3,4,5]; 0.5,0.6)	([3,5,6,7]; 0.4,0.6)	([1,3,4,7]; 0.6,0.8)
Y ₄	([1,2,3,4]; 0.3,0.7)	([1,2,4,5]; 0.3,0.6)	([2,3,4,7]; 0.3,0.4)	([1,2,4,6]; 0.2,0.5)	([2,4,5,6]; 0.4,0.7)

Table 4. The trapezoidal vague number evaluation information of P_3

	B_1	B_2	B_3	B_4	B_5
Y_1	([1,2,4,7]; 0.3,0.9)	([3,4,5,6]; 0.8,0.9)	([2,3,5,8]; 0.3,0.4)	([1,2,4,7]; 0.4,0.7)	([4,5,6,7]; 0.1,0.5)
Y_2	([3,4,5,6]; 0.5,0.6)	([1,2,3,4]; 0.5,0.6)	([2,3,4,6]; 0.1,0.2)	([2,3,4,5]; 0.3,0.4)	([2,3,5,6]; 0.2,0.7)
Y ₃	([1,3,4,5]; 0.4,0.8)	([2,3,5,7]; 0.3,0.7)	([2,3,5,7]; 0.5,0.6)	([1,3,6,7]; 0.2,0.7)	([1,3,6,8]; 0.7,0.9)
Y_4	([2,3,4,5]; 0.2,0.7)	([1,2,4,5]; 0.2,0.5)	([1,2,5,6]; 0.2,0.5)	([4,5,6,8]; 0.6,0.9)	([1,2,4,6]; 0.3,0.6)

For the 4 coal mines of Datong coal mine group according to the 5 indices, each expert weight can be calculated by formula (7) and (8). Comprehensive trapezoidal

vague number information is obtained by aggregating the trapezoidal vague number information which is given by each expert through formula (6).

Calculate it according to trapezoidal vague number score value function, the results are shown in Table 5.

	B_1	B_2	B_3	B_4	B_5
Y_1	-0.652	-0.913	1.823	-0.691	-2.9
Y_2	1.4	2.06	-0.048	-0.45	-0.084
Y_3	1.771	3.474	2.629	1.561	1.219
Y_4	-0.366	-1.569	1.753	1.908	0.924

Table 5. Comprehensive trapezoidal vague number score function value of each index of coal mine

Index weight vector Wa is obtained by formula (10). The score of trapezoidal vague number of each coal mine emergency rescue capability can be obtained by weighted sum of the scores of each index. They are S(Y1)=-0.664, S(Y2)=0.578, S(Y3)=2.129, S(Y4)=0.529.

According to the principle of high score corresponding to optimal case, sort as $Y_3 \succ Y_2 \succ Y_4 \succ Y_1$, that means the coal mine emergency rescue capability of Y_3 is the best.

4.2. Decision ranking by trapezoidal vague number utility value method.

Base on the data in Table 5, the utility values of factors of each coal mine can be obtained by using the utility function of trapezoidal vague number. See Table 6.

Table 6. Comprehensive trapezoidal vague number utility function value of Each index of coal mine

	B_1	B_2	B_3	B_4	B_5
Y_1	0.382	0.348	0.404	0.365	0.220
Y_2	0.430	0.496	0.238	0.376	0.434
Y_3	0.680	0.500	0.470	0.512	0.405
Y_4	0.352	0.322	0.660	0.333	0.413

As we know, the index weight vector is Wa. The utility value of trapezoidal vague number of each coal mine emergency rescue capability can be obtained by weighted sum of the utility value of each index. They are m(Y1)=-0.664, m(Y2)=0.578, m(Y3)=2.129, m(Y4)=0.529.

According to the principle of high utility value corresponding to optimal case, sort as $Y_3 \succ Y_4 \succ Y_2 \succ Y_1$, that means the coal mine emergency rescue capability of Y_3 is the best.

4.3. Decision ranking by VIKOR method of trapezoidal vague numbers.

Then we use formula (12) and (13) calculate group utility value S_i and individual regret value and individual regret value R_i of every coal mine respectively. $S_1 = 1.175$, $S_2 = 0.518$, $S_3 = 0.3$, $S_4 = 0.928$, $R_1 = 0.494$, $R_2 = 0.136$, $R_3 = 0.125$, $R_4 = 0.450$.

After that, the benefit ratio Q_i of each coal mine can be obtained by formula (14), where Q denotes the coefficient of decision making mechanism, we take Q=0.5.

The ascending order of the utility value, the individual regret value and the benefit ratio is $S_3 < S_2 < S_4 < S_1$, $R_3 < R_2 < R_4 < R_1$, According to the principle of small value corresponding to optimal case, we can get the ascending order of the four coal mines that is $Y_3 \succ Y_2 \succ Y_4 \succ Y_1$, i.e., the emergency rescue capability of Y_3 is the best.

5. Results analysis

The above three methods were carried out respectively to evaluate the emergency rescue capability of four coal mines of Datong Coal Mine Group. The comparison curves of calculation results are as shown in Figure 1.Through analysis we can know that in the three decision making methods, the emergency rescue capacity of the best coal mine and the worst coal mine have not changed, i.e., Y_3 is the best while Y_1 is the worst. Furthermore, since the ranking results are the same based on the trapezoidal vague number score method and the VIKOR method, we can draw that the order of emergency rescue capability of the four coal mine of Datong Coal Mine Group is $Y_3 \succ Y_2 \succ Y_4 \succ Y_1$.



Fig. 1. The evaluation results contrast curves of three models

6. Conclusion

The results show that the emergency rescue capacity obtained by the three decision models is the strongest and the worst coal mine is the same. At the same time, the coal mine emergency rescue capacities ranking based on the trapezoidal vague number score and VIKOR method are the same. The research provides a reference for government departments and relevant understanding of the coal mine emergency rescue capability of Datong Coal Mine Group. At last, we applied the three group decision in the four coal mines of Datong Coal Mine Group. The results show that the strongest and the worst emergency rescue capacities of coal mine obtained by the three decision models are the same. At the same time, the coal mine emergency rescue capacities ranking based on the trapezoidal vague number score and VIKOR method are the same.

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